Lecturered By Dr. H. K. Yadav
Deptt. of Mathematics
Marwari College, Darbhanga

B.Sc. part - III (H), paper - III, Date - 21-04-2020

Differential Calculus (Tangents and Normals)

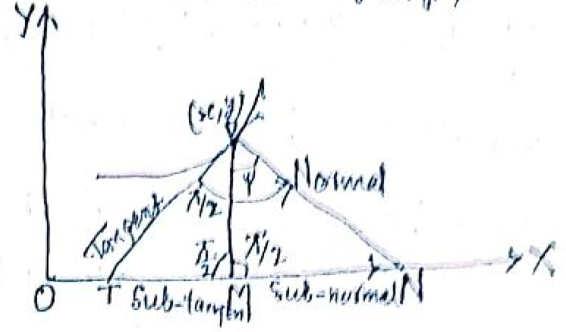
Q.1. Prove that (I) the Cartesian sub-tangent $= \frac{y}{y_1}$

(II) the Cartesian sub-normal $= yy_1$

(III) the length of the tangent $= \frac{y}{y_1}\sqrt{1+y_1^2}$

and (IV) the length of the normal $= y\sqrt{1+y_1^2}$, where $y_1 = \frac{dy}{dx}$

Soln:



Let the eqn of the curve be $y = f(x)$

Let $P(x,y)$ be any point on it. Let PM, PT, PN be the ordinate, tangent and normal to the curve and let PT make an angle ψ with the axis of x, then $\tan\psi = \frac{dy}{dx}$.

Let the tangent at P meet the x-axis in T. Then the length TM is called the Sub-tangent at P and the length MN is called the Sub-normal at P.

In the right angle triangle PTM and PNM

Since $\angle MPN = \angle PTN = \psi$ and $PM = y$

(I) Sub tangent $= TM = y\cot\psi = \frac{y}{\tan\psi} = \frac{y}{\frac{dy}{dx}} = \frac{y}{y_1}$

II Sub normal $= MN = y\tan\psi = y\frac{dy}{dx} = yy_1$

III Tangent $= PT = y\,\text{cosec}\,\psi = y\sqrt{1+\cot^2\psi}$

$= y\,\frac{\sqrt{1+\tan^2\psi}}{\tan\psi} = y.\frac{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\frac{dy}{dx}} = \frac{y}{y_1}\sqrt{1+y_1^2}$

IV Normal $= PN = y\sec\psi = y\sqrt{1+\tan^2\psi} = y\sqrt{1+\left(\frac{dy}{dx}\right)^2} = y\sqrt{1+y_1^2}$

where $y_1 = \frac{dy}{dx}$.

B.Sc-part-II, paper-III. Date 24-04-2020.

Q.2. Prove that the eqn of the normal to the curve $y=f(x)$ at the point (x,y) is $(X-x)+(Y-y)\frac{dy}{dx}=0$ and that to the curve $f(x,y)=0$ at (x,y) is $\frac{X-x}{f_x}=\frac{Y-y}{f_y}$.

Soln: We know that by the normal to a curve at a point we mean the straight line which passes through that point and is at right angles to the tangent at the point.

The equation of the tangent to the curve $y=f(x)$ at a point (x,y) is

$$Y-y=\frac{dy}{dx}(X-x), \text{ where slope is } \frac{dy}{dx}$$

$\therefore$ the slope of the normal $= -\frac{1}{\frac{dy}{dx}}$

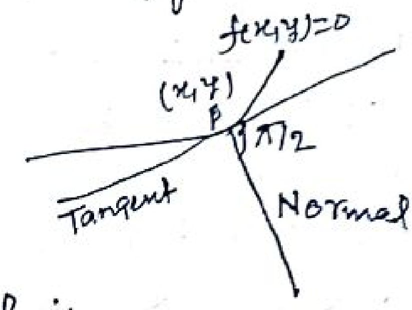
Hence, the eqn to the normal at the point (x,y) is

$$Y-y=\frac{1}{-\frac{dy}{dx}}(X-x)$$

$$\Rightarrow (X-x)+(Y-y)\frac{dy}{dx}=0$$

If the eqn of the curve be $f(x,y)=0$, then the eqn of the tangent is $(X-x)f_x+(Y-y)f_y=0$

whose slope is $-\frac{f_x}{f_y}$



Therefore, the slope of the normal is $\frac{f_y}{f_x}$

Hence, the eqn of the normal is

$$Y-y=\frac{f_y}{f_x}(X-x)$$

$$\Rightarrow \frac{X-x}{f_x}=\frac{Y-y}{f_y}$$

Proved.

Scanned with CamScanner

Lecturered By Dr. H. K. Yadav.
Deptt. of Mathematics
Marwari College, Darbhanga. Date-24-04-2020

Set theory. B.Sc-part-I(H), paper-I

Q.1. Define countable and uncountable sets and prove that the set of rational numbers is countable.

Soln Countable and uncountable sets:—

Let X be any set can be mapped one-one onto the set $\{1, \cdots, n\}$, where $n \in N$, then X is called a finite set, we also say that X has n elements.

In this case, if $X = \{1, \cdots, n\}$ is established by function f and $1 \le k \le n$, then under f, the image of $k \in \{1, \cdots, n\}$ is denoted as f_k. Hence, we write $X = \{f_1, \cdots, f_n\}$. The empty set ϕ is taken as finite.

Here, the ordered n-tuple $\langle f_1, \cdots, f_n \rangle$ is merely a permutation of X. In all, there are $\underline{|n}$ permutations.

The members of $N = \{1, 2, 3, 4, \cdots, n, \cdots\}$ are called finite natural numbers.

A set X which is not finite is called an infinite set. From the above discussion, it will be clear that X will be infinite only when there is no one-one map of X onto $\{1, \cdots, n\}$ for any n.

Proof: Let $A_n = \{\frac{0}{n}, -\frac{1}{n}, \frac{1}{n}, -\frac{2}{n}, \frac{2}{n}, \cdots\}$

So that A_n is the set of all those rationals whose denominator is $n \in N$. Here, it is clear that

$$f : N \to A_n$$

defined by $f(x) = \begin{cases} \frac{x-1}{2n}, & \text{when } x \text{ is odd.} \\ -\frac{x}{2n}, & \text{when } x \text{ is even} \end{cases}$

is one-one onto mapping. Thus each A_n is countable.

Now, $Q = \cup A_n$ is a countable union of countable sets and hence it is countable.

Thus, the set of rational numbers is countable.

Set theory

Q.2. Prove that the set of all irrational numbers is uncountable.

proof:- Since the set R of all real numbers and the set Q of rational numbers is countable. then if B is a countable subset of an uncountable set A, then A-B is uncountable. ~~It follows that~~ The complement of the set of rationals relative to the set of real numbers i.e. R-Q of irrational numbers is uncountable.

Q.3. Prove that the set A of algebraic numbers is countable enumerable.

Proof: Let $P = \cup \{P_n(x) : P_n(x) = 0,\ n \in N\}$, where $P_n(x)$ stands for a polynomial of degree n with integral coefficients, is enumerable. When

$$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_mx^m$$

with integral coefficients is enumerable.

Since, an algebraic number is the root of a polynomial $P(x) = 0$ with integral coefficients. Let us consider $A_n = \{x : x$ is a solution of $P_n(x) = 0\}$

Since, a polynomial of degree n has at most n roots and hence A_n is finite.

Now, $A = \cup \{A_n : n \in N\}$ is the ~~set~~ of algebraic numbers and it is union of countable ~~sets~~ and hence A, the set of all algebraic numbers is countable.

Q.4. Prove that the set of all irrational numbers is uncountable. the set of all real numbers ~~and~~ is uncountable-

Proof: Since the set R of all real numbers is uncountable because of the set of unit interval [0,1] is uncountable. when the set of rational numbers is countable and the set of all positive rationals is countable. Then if B is a countable subset of an uncountable set A, then A-B is uncountable. The complement of the set of rationals relative to the set of real numbers. i.e. R-Q of irrational numbers is uncountable.